

# GCSE Maths

Foundation and Higher  
Weeks 9-12

## Online Tuition Workbook

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[Week 2 \(Number\) - Percentages](#)

[Week 3 \(Algebra\) - Algebraic Fundamentals](#)

[Week 4 \(Algebra\) - Factorising / Sequences](#)

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# Week 9

## Unit: Geometry

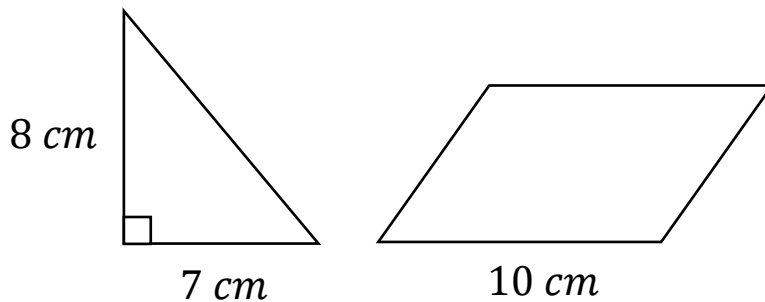
### Areas and Perimeters of 2D shapes

- Calculate the areas and perimeters of various 2D shapes
- Calculate the area and circumference of circles
- Calculate the area and perimeter of sectors

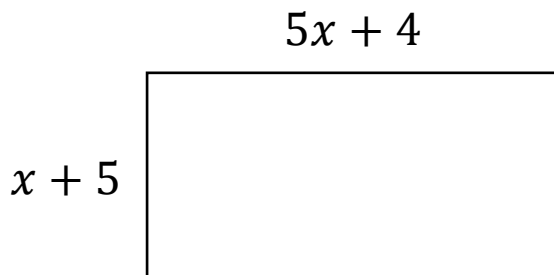
1. A square has an area of  $36 \text{ cm}^2$ , what is the value of its perimeter?

2. The triangle and parallelogram have equal areas.

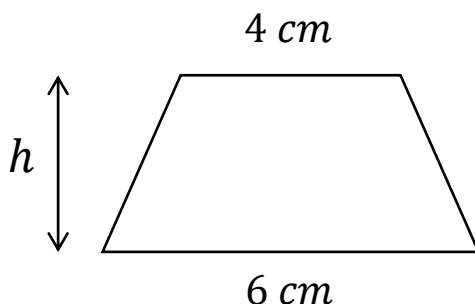
Find the perpendicular height of the parallelogram



3. The length of the rectangle is double its width. Calculate the area.

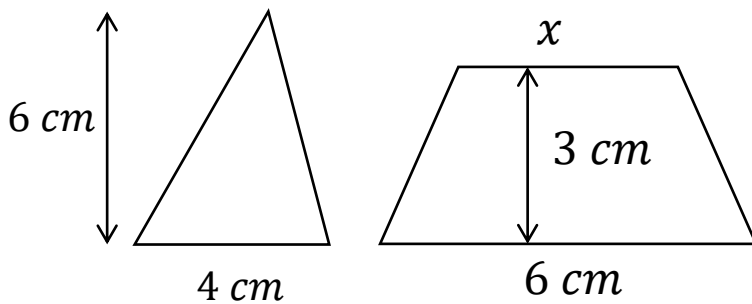


4. The trapezium has an area of  $60 \text{ cm}^2$ . Find the value of  $h$

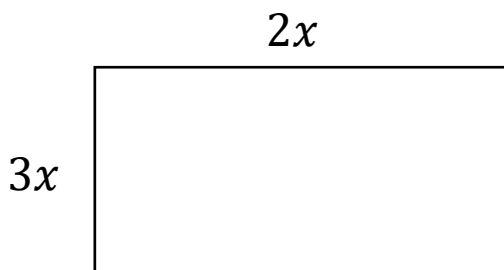


1. A square has perimeter of  $20\text{cm}$ , what is the value of its area?

2. The triangle and trapezium have equal areas.  
Find the value of  $x$

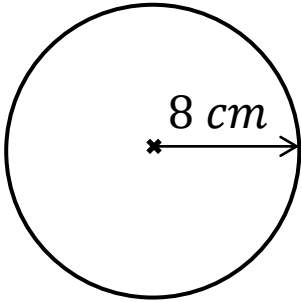


3. The area of the rectangle is  $294\text{ cm}^2$ . Calculate its perimeter.



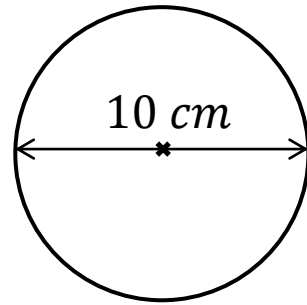
## Worked Example

Calculate the area and circumference of the circle

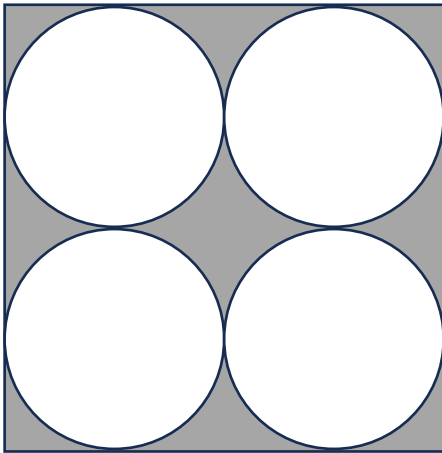


## One to try

Calculate the area and circumference of the circle



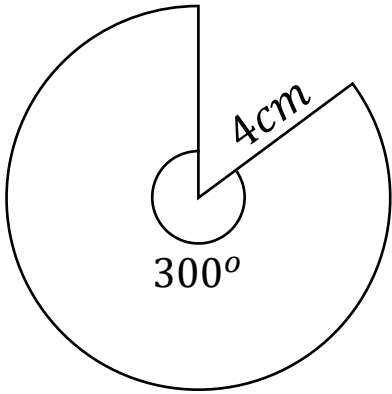
1. Find the circumference of a circle that has an area of  $40 \text{ cm}^2$   
Give your answer to 1 decimal place
  
2. Find the area of a circle that has a circumference of  $30 \text{ cm}$   
Give your answer to 2 decimal places
  
3. Each circle has a diameter of  $20 \text{ cm}$ .  
Find the area of the shaded region



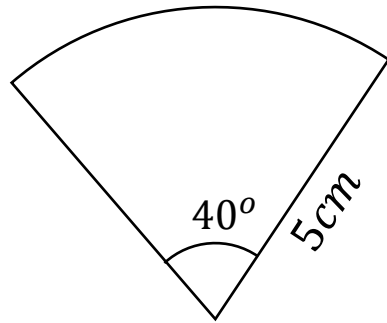
Worked Example

One to try

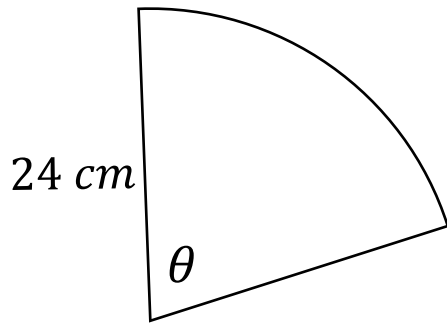
Calculate the area and perimeter of this sector



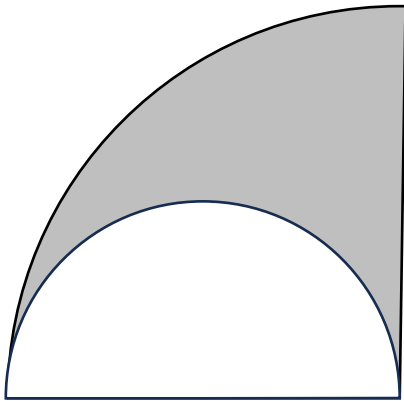
Calculate the area and perimeter of this sector



1. The area of the sector is  $200\text{cm}^2$ . Find the angle  $\theta$  to 1 decimal place

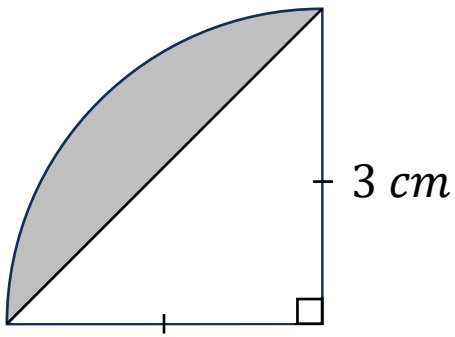


2. The area of the semi-circle is  $18\pi\text{ mm}^2$ .  
Calculate the area of the shaded section of the quarter circle.

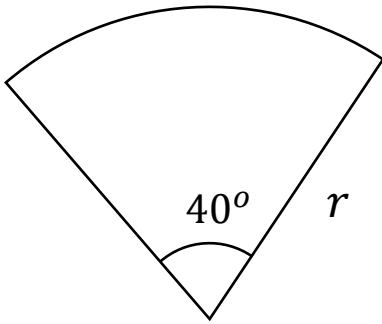




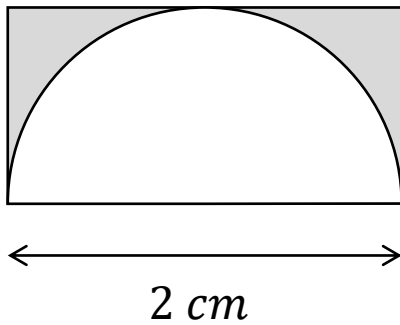
1. Calculate the exact area of the shaded region



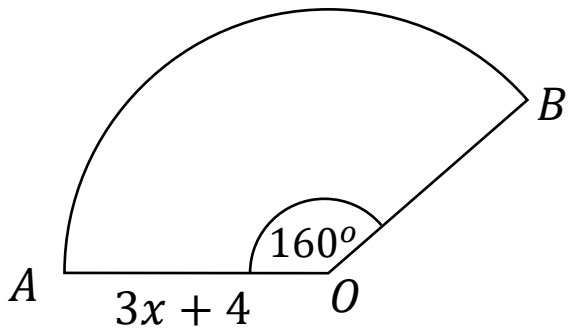
2. The area of the sector is  $15\text{ cm}^2$ . Find the radius, to 1 decimal place



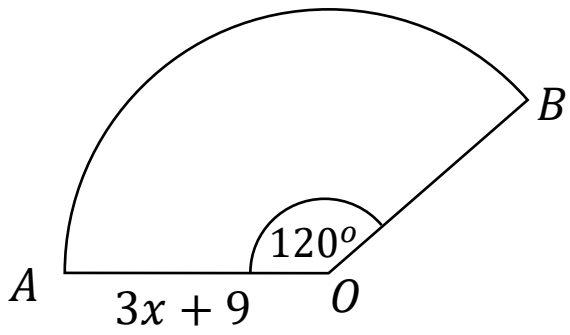
3. Calculate the exact area of the shaded region



Write a fully simplified expression for the area of sector  $AOB$



Write a fully simplified expression for the perimeter of sector  $AOB$



# Week 10

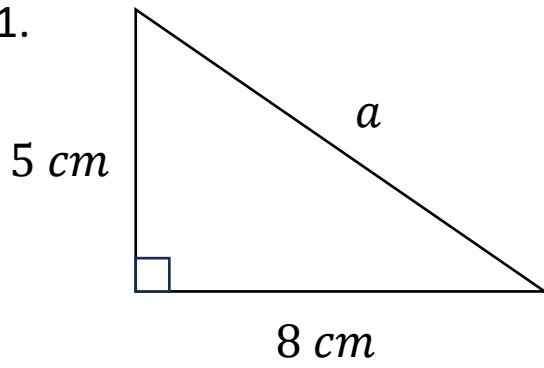
## Unit: Geometry

### Working with Right-angled Triangles

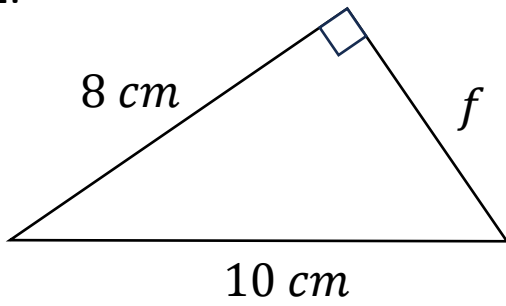
- Use Pythagoras' Theorem
- Find missing angles and lengths using right-angled trigonometry
- Derive and use exact trigonometric values

Find the value of each of the unknowns

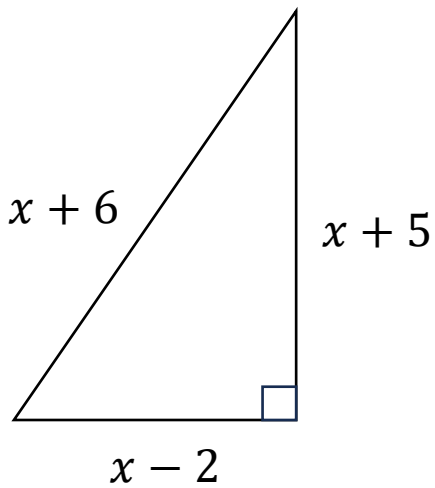
1.



2.

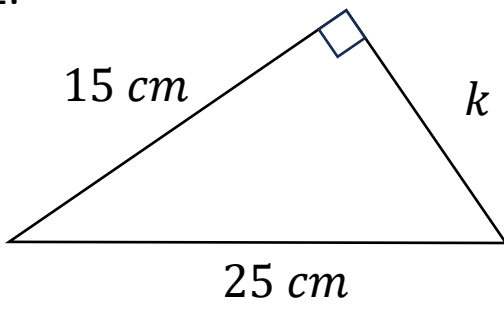


3.

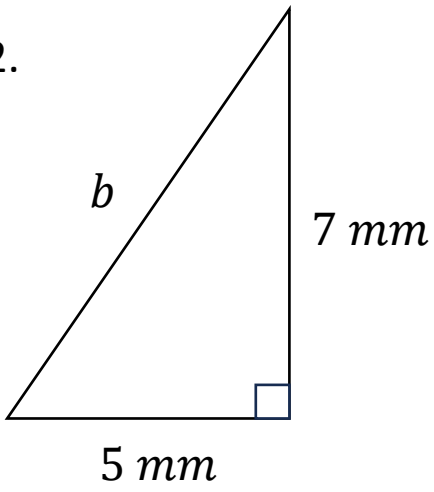


Find the value of each of the unknowns

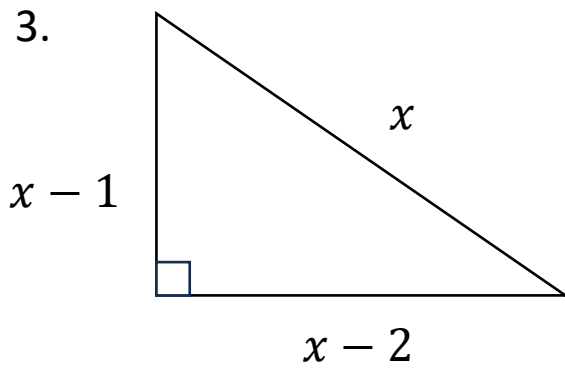
1.



2.



3.



A triangle has side lengths of  $20\text{ cm}$ ,  $21\text{ cm}$  and  $29\text{ cm}$

Does the triangle have a right angle? Show your working out.

A triangle has side lengths of  $10\text{ cm}$ ,  $11\text{ cm}$  and  $13\text{ cm}$

Does the triangle have a right angle? Show your working out.

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A triangle has side lengths of  $7\text{ cm}$ ,  $8\text{ cm}$  and  $10\text{ cm}$

Does the triangle have a right angle? Show your working out.

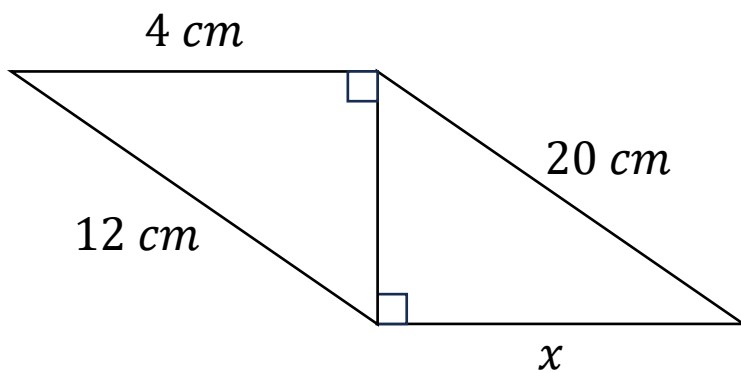
A triangle has side lengths of  $21\text{ cm}$ ,  $28\text{ cm}$  and  $35\text{ cm}$

Does the triangle have a right angle? Show your working out.

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1. Calculate the exact value of  $x$



2. A ship leaves point A and sails for  $3.5\text{ km}$  due North.

The ship then sails for  $4.5\text{ km}$  due West to reach point B.

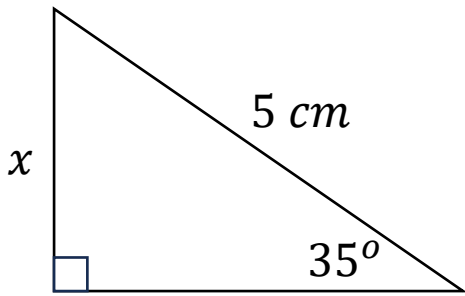
Calculate the shortest distance between point A and point B (to 1dp)

3. A square has a perimeter of  $8\text{ cm}$ .

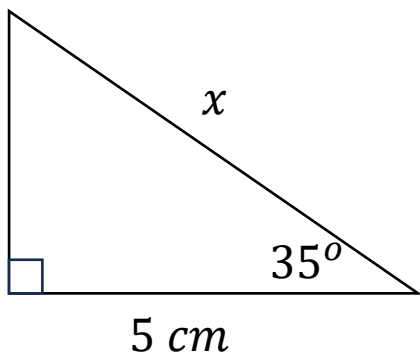
Find the exact length of its diagonal.

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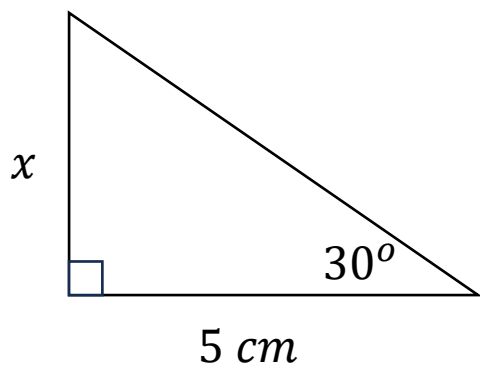
Find the length of the missing side



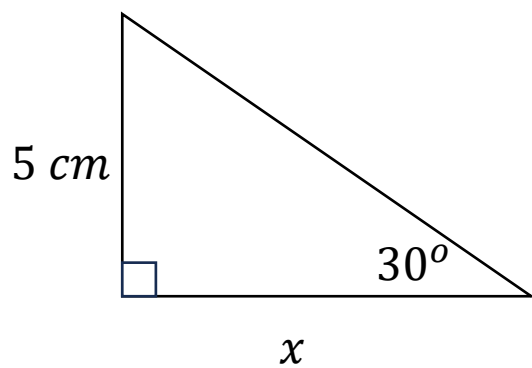
Find the length of the missing side



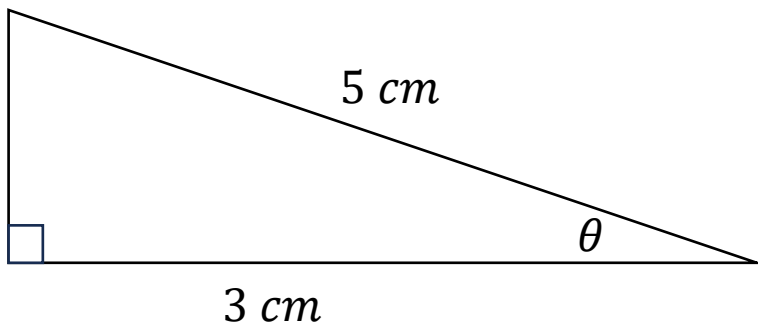
Find the length of the missing side



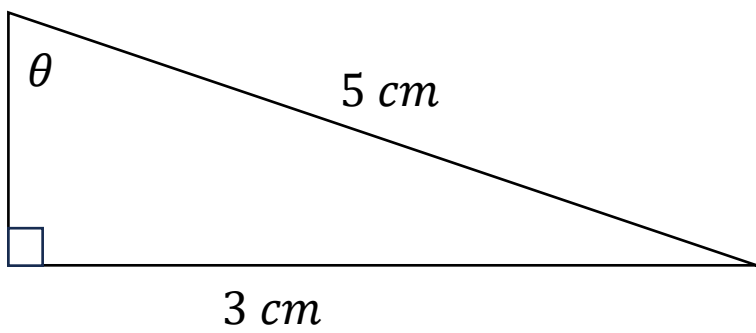
Find the length of the missing side



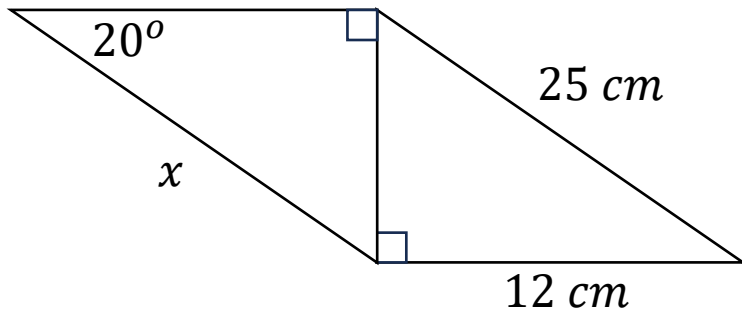
Find the value of  $\theta$  to 1 decimal place



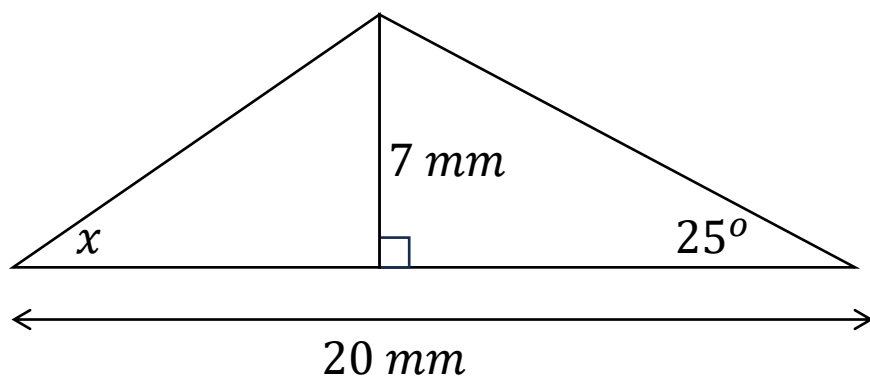
Find the value of  $\theta$  to 1 decimal place



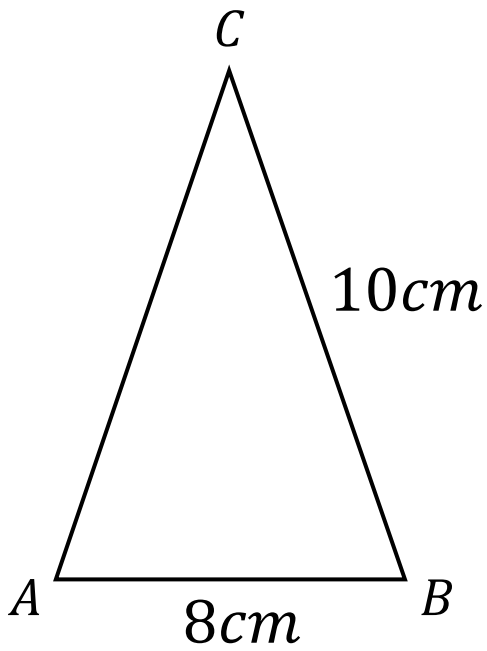
Find the value of  $x$  to 1 decimal place



Find the value of angle  $x$  to 1 decimal place.

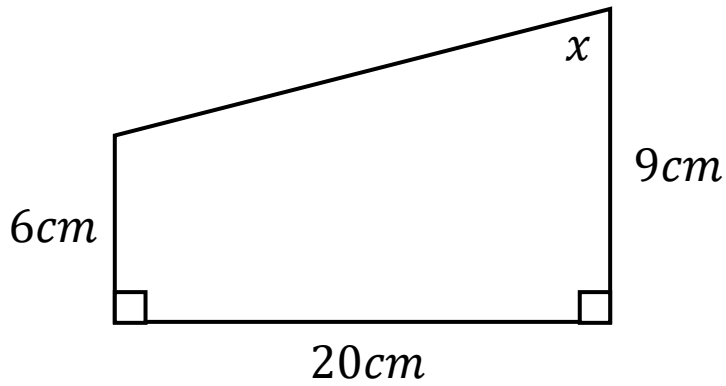


1. Triangle  $ABC$  is isosceles. Work out angle  $CAB$  and angle  $BCA$



2. Find the perpendicular height of an equilateral triangle of side  $4\text{cm}$

1. Find the size of angle  $x$  to 1 decimal place



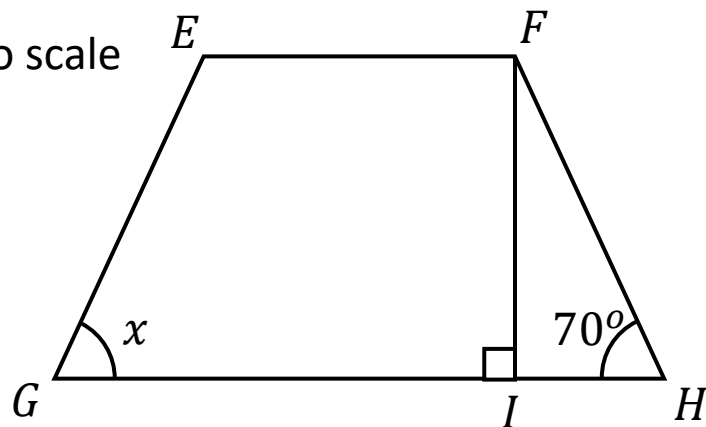
2.  $EFGH$  is a trapezium, not draw to scale

$$GH = 25\text{ cm}$$

$$IH = 4\text{ cm}$$

$$EF = 14.5\text{ cm}$$

Find the size of angle  $EGI$





1. Write down the exact value of  $\sin(60^\circ)$

2. Write down the exact value of  $\cos(45^\circ)$

3. Write down the exact value of  $\sin(0)$

4. Write down the exact value of  $\cos(0)$

5. Write down the exact value of  $\tan(30^\circ)$

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1. Write down the exact value of  $\sin(30^\circ)$

2. Write down the exact value of  $\sin(45^\circ)$

3. Write down the exact value of  $\tan(0)$

4. Write down the exact value of  $\cos(60)$

5. Write down the exact value of  $\tan(45^\circ)$

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# Week 11

## Unit: Geometry

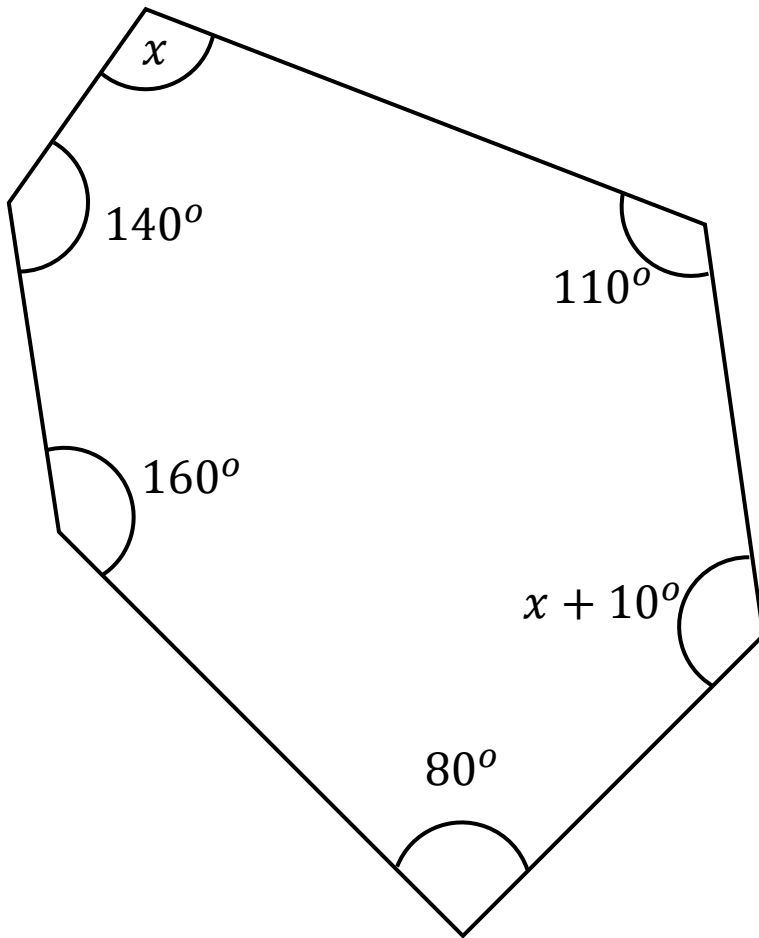
### Angles in Polygons and Parallel Lines

- Calculate exterior and interior angles in regular polygons
- Calculate exterior and interior angles in irregular polygons
- Find missing alternate, corresponding and co-interior angles

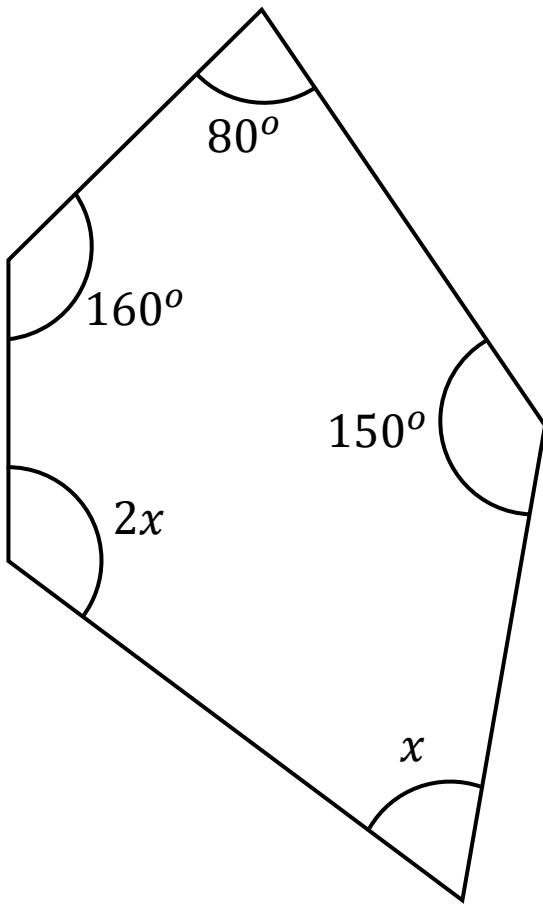
1. What is the size of one exterior angle in a regular octagon?
  2. What is the sum of interior angles in a hexagon?
  3. The size of each exterior angle in a regular polygon is  $15^{\circ}$   
How many sides does the polygon have?
  4. What is the size of an interior angle in a regular 20 - sided shape?
  5. What is the size of an exterior angle in a regular 12 - sided shape?
  6. The size of an interior angle in a regular polygon is  $160^{\circ}$   
How many sides does the polygon have?
-

1. What is the size of one exterior angle in a regular pentagon?
  2. What is the sum of interior angles in a decagon?
  3. The size of each exterior angle in a regular polygon is  $30^\circ$   
How many sides does the polygon have?
  4. What is the size of an interior angle in a regular 15 - sided shape?
  5. What is the size of an exterior angle in a regular nonagon?
  6. The size of an interior angle in a regular polygon is  $170^\circ$   
How many sides does the polygon have?
-

Find the value of  $x$



Find the value of  $x$



An irregular octagon has four interior angles of size  $2x$ , and another four interior angles of size  $3x$ .

Find the value of  $x$ .

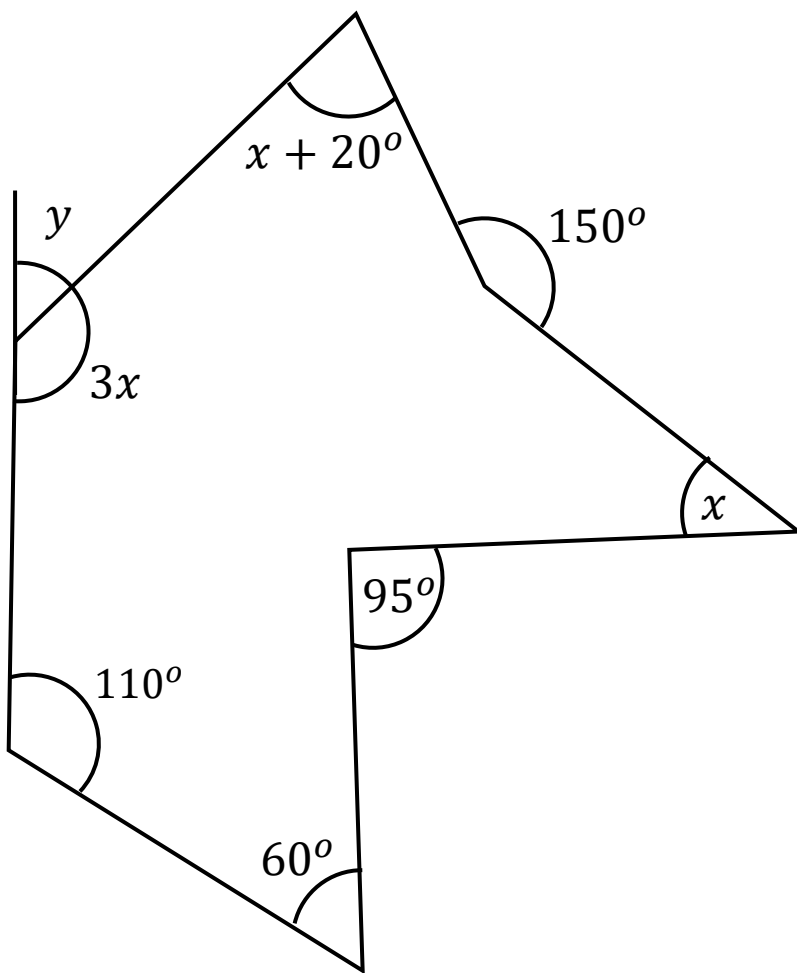
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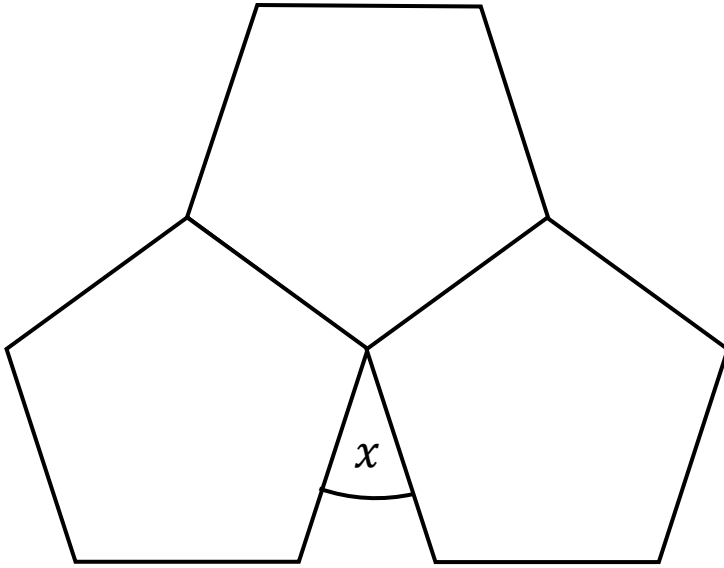
An irregular nonagon has three interior angles of size  $x^\circ$  and another three interior angles of  $3x + 10^\circ$ . The remaining interior angles are of size  $2x$ . Find the value of  $x$ .

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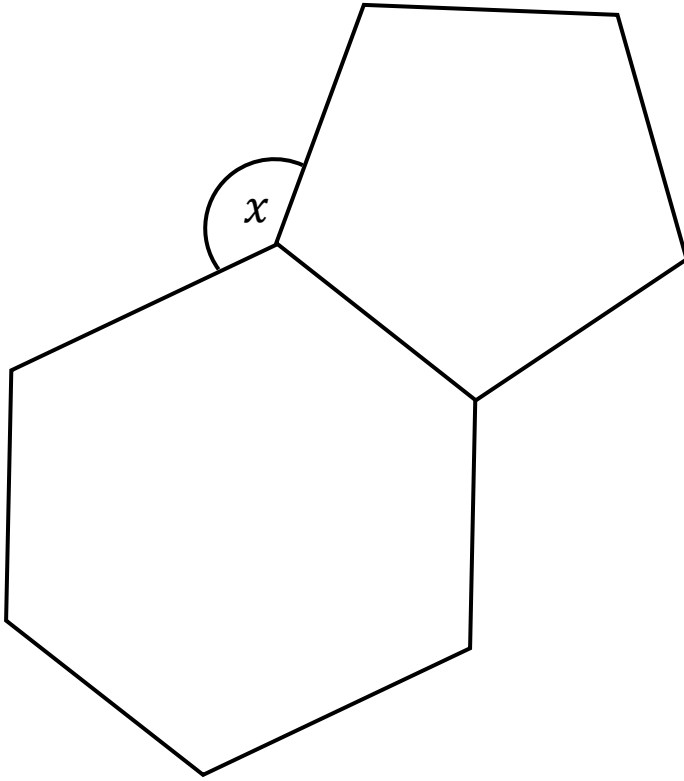
Find the value of  $x$  and the size of angle  $y$



The diagram shows three regular pentagons meeting at a point. Find the size of the angle marked  $x$



The diagram shows a regular pentagon and a regular hexagon  
Find the size of the angle marked  $x$

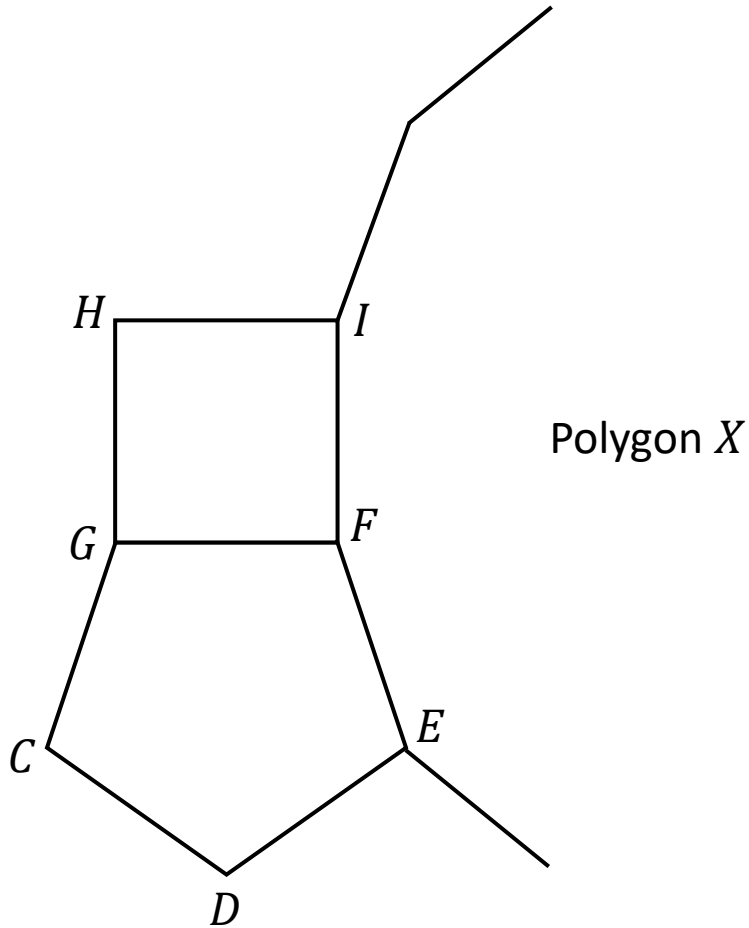


$CDEFG$  is a regular pentagon

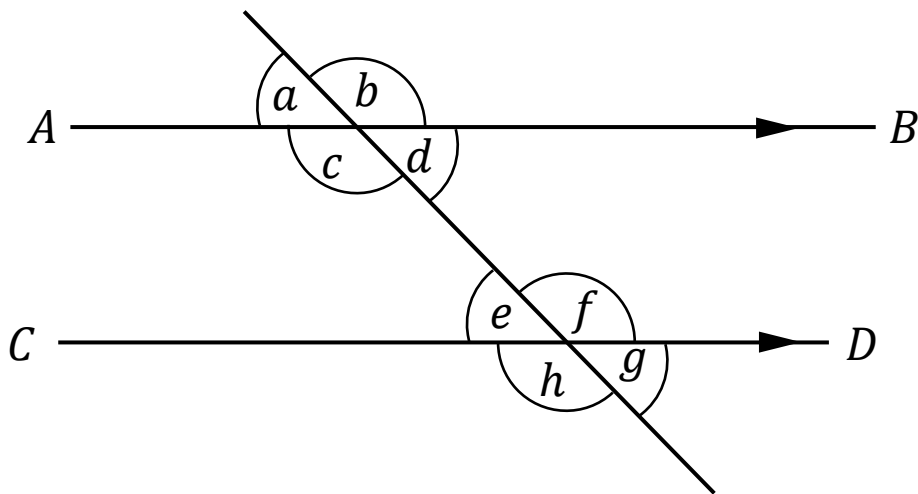
$FGHI$  is a square

$FE$  and  $FI$  are both sides of another regular polygon  $X$

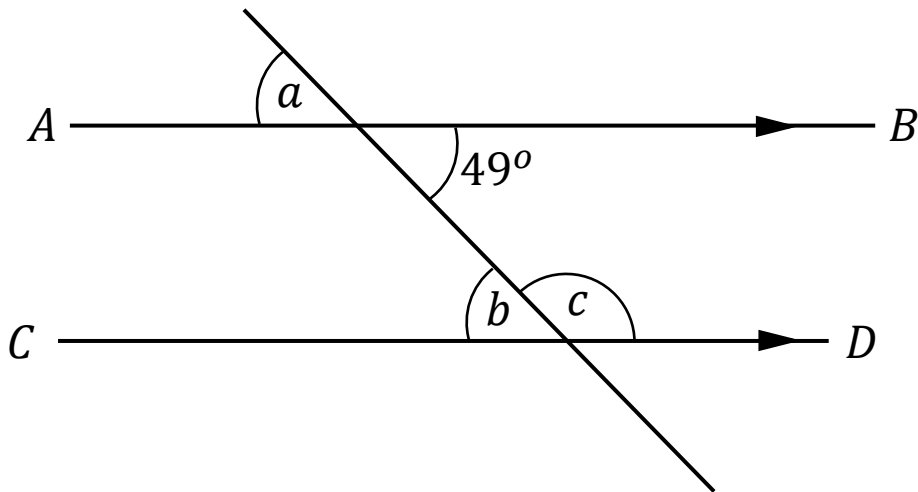
How many sides does polygon  $X$  have?



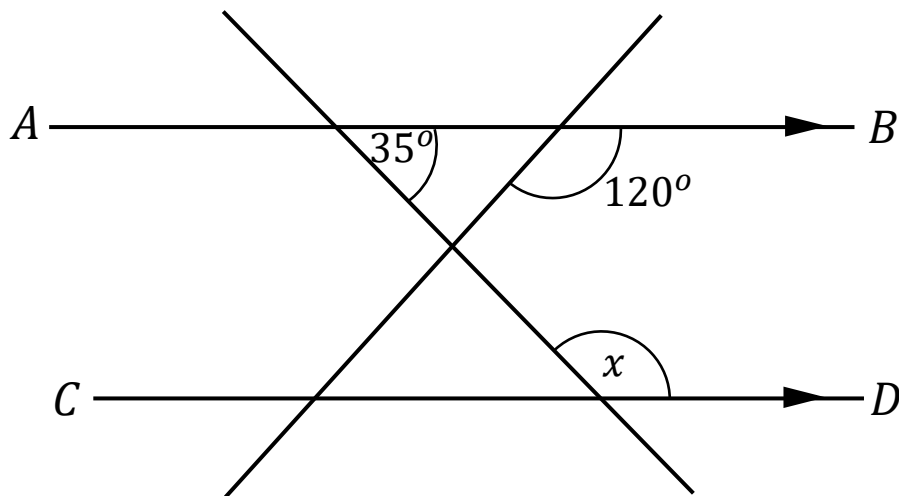
- i) Name all pairs of corresponding angles
- ii) Name all pairs of alternate angles
- iii) Name all pairs of co-interior angles
- iv) Name all pairs of vertically opposite angles



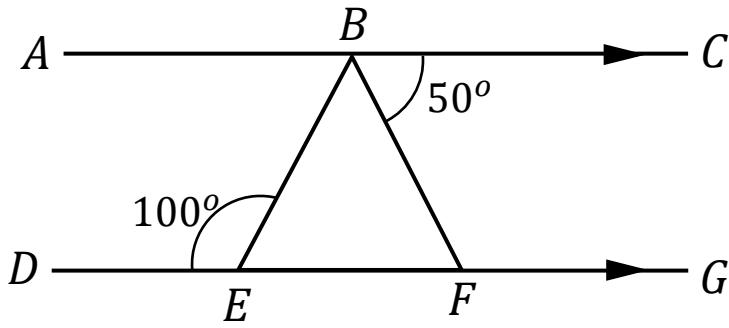
Find the size of angle  $a$ , angle  $b$  and angle  $c$



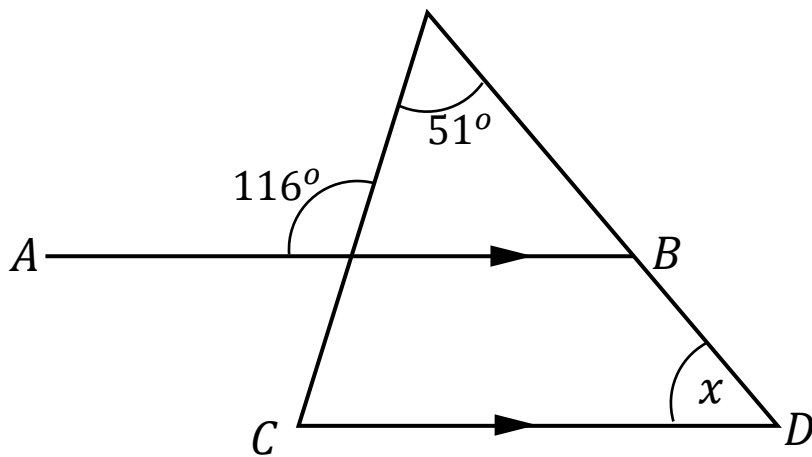
Find the size of angle  $x$



1. Show that triangle  $BEF$  is isosceles

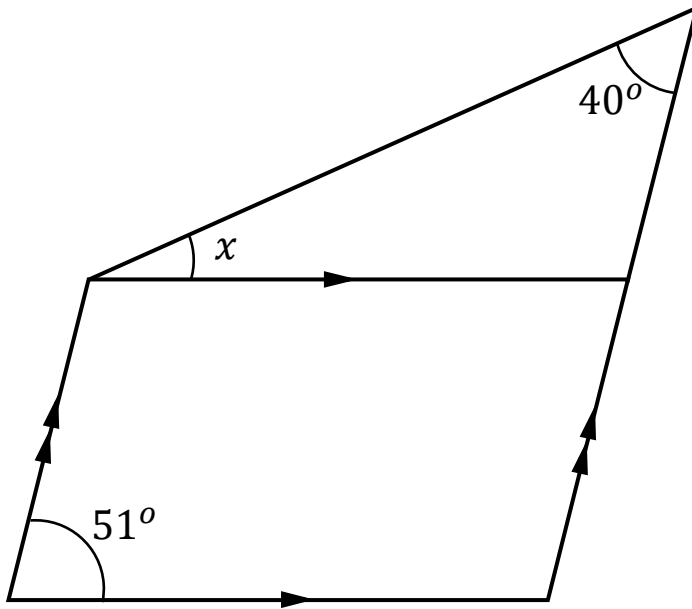


2. Find the size of angle  $x$





Find the size of angle  $x$



# Week 12

## Unit: Probability

### Calculating Probabilities

- Calculate the probability of combined events
- Fill in, and calculate probabilities from, two-way tables
- Fill in, and calculate probabilities from, Venn diagrams

## Worked Example

## One to try

In a bag of counters there are only:  
3 red counters  
5 blue counters  
4 green counters

A counter is picked out and replaced.  
A second counter is then taken.  
Calculate the probability of:

- i) Picking two red counters in a row
- ii) Picking a red counter then a blue counter
- iii) Not picking a green counter twice in a row

In a bag of marbles there are only:  
6 black marbles  
12 red marbles  
2 white marbles

A marble is picked and not put back.  
A second marble is then taken.  
Calculate the probability of:

- i) Picking two black marbles in a row
- ii) Picking a red marble then a white marble
- iii) Not picking a black marble twice in a row

1. There are red, yellow and blue pens in a bag

A pen is picked at random

The table shows the probability of picking each colour

i) Complete the probability table

ii) There are 30 blue pens in the box, how many yellow pens are there?

Colour	Red	Blue	Yellow
Probability	$\frac{1}{2}$	$\frac{1}{5}$	

2. There are green, orange, purple and black pens in a bag

A pen is picked at random

The table shows the probability of picking each colour

The probability of picking a green, purple or black pen is equal

i) Complete the probability table

ii) There are 300 pens in the box. How many purple pens are there?

Colour	Green	Orange	Purple	Black
Probability		0.55		

1. There are black, white and grey counters in a bag  
A counter is picked at random  
The table shows the probability of picking each colour  
The probability of picking a white counter is 4 times that of grey

- i) Complete the probability table  
ii) A counter is picked and put back. This is done 80 times.  
How many black counters would you expect to pick out?

<b>Colour</b>	Black	White	Grey
<b>Probability</b>	0.4		

2. There are red, blue, yellow and black counters in a bag  
A counter is picked at random  
The table shows the probability of picking each colour  
Find the value of  $x$

<b>Colour</b>	Red	Blue	Yellow	Black
<b>Probability</b>	$x$	$x + 0.1$	0.3	$2x$

200 year 8 students were surveyed on their sport choice for PE  
They could either play football, cricket or badminton  
90 of the students are boys  
70 students chose cricket  
20 of the 42 students that chose football were boys  
50 girls chose badminton  
Complete the two-way table

	Football	Cricket	Badminton	Total
Boys				
Girls				
Total				

- i) What is the probability that a student chose badminton?
- ii) What is the probability that a student chose football, given they were a girl

A cricket team played 50 games

In the 25 home game that were played, they drew 7

5 of the 15 games they lost were away

They won 6 away games

Complete the two-way table

	Win	Lose	Draw	Total
Home				
Away				
Total				

What is the probability that they drew, given that it was an away game?

1. 80 students were surveyed:

50 have a brother

40 have a sister

20 have both

Construct a Venn diagram

Find the probability that a student has neither a brother or sister

2. 60 people were surveyed:

40 have been to France

30 have been to Spain

5 have been to neither

Construct a Venn Diagram

Find the probability of someone having been to France, given they have been to Spain



1. 100 students were surveyed:
  - 50 liked football
  - 30 liked football and cricket
  - 20 liked neither

Construct a Venn diagram

Find the probability of a student liking football given they liked cricket

2. A group of children were surveyed:
  - 20% played the piano
  - 25% played the guitar
  - 60% played neither

Construct a Venn diagram

Find the probability that a student plays both the piano and guitar.

70 people were surveyed on which juices they like.

The three options were apple, orange and cranberry.

Everyone liked at least one type of juice.

21 people like all three juices.

18 people like orange and apple but do not like cranberry.

30 people like apple and cranberry.

50 people like orange.

23 people like orange and cranberry.

3 people like only cranberry.

A person is selected at random.

Work out the probability that this person likes apple juice.

50 people were surveyed on which sports they liked

The three options were football, cricket and rugby

2 people didn't like any sport.

10 people like all three sports.

15 people like cricket and football.

20 people like football.

35 people like rugby.

23 people like rugby and cricket.

8 people like only rugby.

A person is selected at random.

Work out the probability this person likes cricket, given they like rugby.