GCSE Maths

Higher Only Content Weeks 9-12

Online Tuition Workbook

Week 5 – The Sine/Cosine Rule

Week 6 - Surds

Week 7 – Direct and Inverse Proportion

Week 8 – Inverse and Composite Functions

Week 9 – Quadratic Sequences / Completing the Square / Factorising Quadratics

Week 10 – Circle Theorems

Week 11 – Vector Proofs

Week 12 – Parallel/Perpendicular Lines and Equations of Tangents

Week 13 – Non-Linear Simultaneous Equations

Week 14 – Probability Equation Questions

Week 15 - Paper 1 Walkthrough

Week 16 - Paper 2/3 Walkthrough

Week 17 - Paper 2/3 Walkthrough

Week 18 - Paper 3 Walkthrough

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Week 9

Quadratic Sequences
Completing the Square
Factorising Quadratics

- Recognise quadratic sequences
- Find the general term of quadratic sequences
- Rewrite quadratic expressions by completing the square
- Identify turning points
- Factorise quadratics where the coefficient of x^2 is $\neq 1$
- Solve and sketch quadratic inequalities

Work out the formula for the n^{th} term of the following quadratic sequence

$$2, \quad 10, \quad 20, \quad 32$$

Work out the formula for the n^{th} term of the following quadratic sequence

$$0, \quad 7, \quad 18, \quad 33$$

Work out the formula for the n^{th} term of the quadratic sequence and hence find the term that has value 424

$$6, \quad 10, \quad 16, \quad 24$$

Write $x^2 - 6x$ in the form $(x + a)^2 + b$ and thus write down the coordinate of the turning point of $y = x^2 - 6x$

Write $x^2 - 3x + 1$ in the form $(x + a)^2 + b$ and thus write down the coordinate of the turning point of $y = x^2 - 3x + 1$

Write $2x^2 - 20x + 3$ in the form $(x + a)^2 + b$ and thus write down the coordinate of the turning point of $y = 2x^2 - 20x + 3$

Write $x^2 - 8x$ in the form $(x + a)^2 + b$ and thus write down the coordinate of the turning point of $y = x^2 - 8x$

Write $x^2 - 7x + 2$ in the form $(x + a)^2 + b$ and thus write down the coordinate of the turning point of $y = x^2 - 7x + 2$

Write $3x^2 - 48x + 1$ in the form $(x + a)^2 + b$ and thus write down the coordinate of the turning point of $y = 3x^2 - 48x + 1$

By completing the square, solve $x^2 + 5x - 2 = 0$ (giving your answer in surd form)

By completing the square, solve $2x^2 + 8x - 1 = 0$ (giving your answer in surd form)

1. Solve by factorising $2x^2 - 8x = 0$

2. Solve by factorising $4x^2 - 25 = 0$

3. Solve by factorising $2x^2 - 98 = 0$

4. Solve by factorising $2x^2 - 3x - 9 = 0$

1. Solve by factorising $3x^2 - x - 10 = 0$

2. Solve by factorising $36x^2 - 25 = 0$

3. Solve by factorising $10x^2 - 90 = 0$

4. Solve by factorising $4x^2 - 12x + 5 > 0$

5. Solve by factorising $2x^2 + 13x + 15 < 0$

Simplify fully $\frac{6x-2}{x^2+6x+5} \div \frac{3x^2+11x-4}{2x^2+13x+15}$

Solve $\frac{12}{x+2} - \frac{2}{x+1} = 3$

Simplify fully $\frac{2x^2+7x+6}{x^2-4} \times \frac{x^2+7x-18}{2x^2-x-6}$

Solve $\frac{7}{x+1} + \frac{4}{2x-9} = 1$

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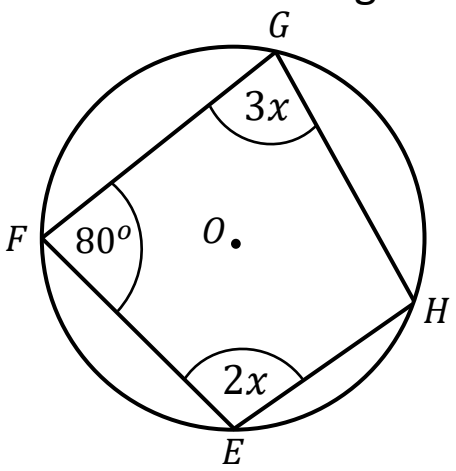
Week 10

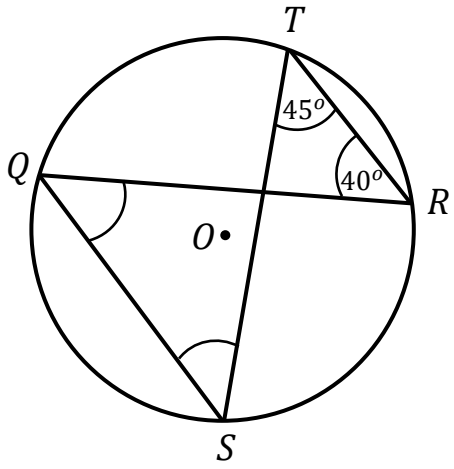
Circle Theorems

- Solve multi-step problems using circle theorems
- Prove circle theorems

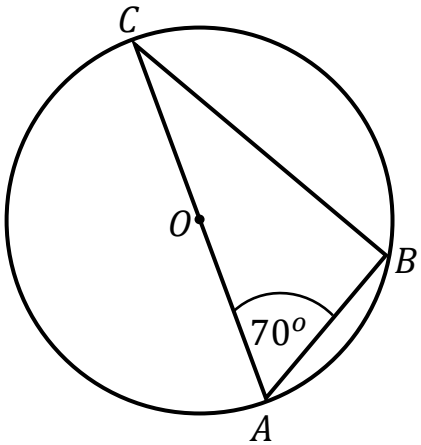
E, F, G and H are points on the circumference of the circle
Work out the size of angle EHG and the value of x



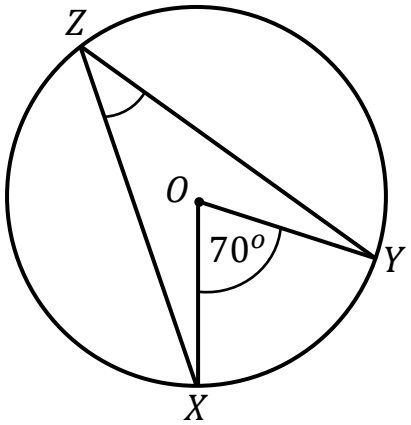
Q, T, R and S are points on the circumference of the circle
Work out the size of angle SQR and angle QST



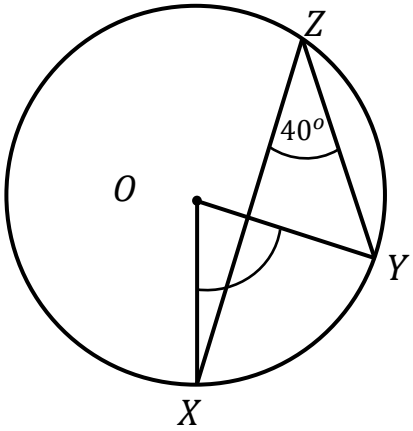
A, C and B are points on the circumference of the circle
AOC is a diameter of the circle
Work out the size of angle ACB



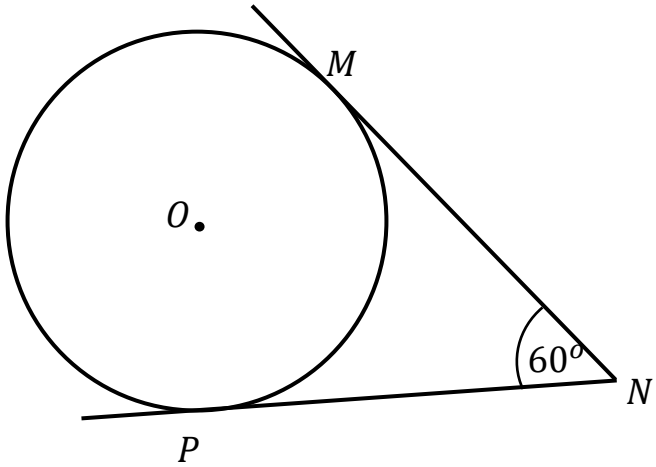
Z, X and Y are points on the circumference of the circle, centre O
Work out the size of angle XZY



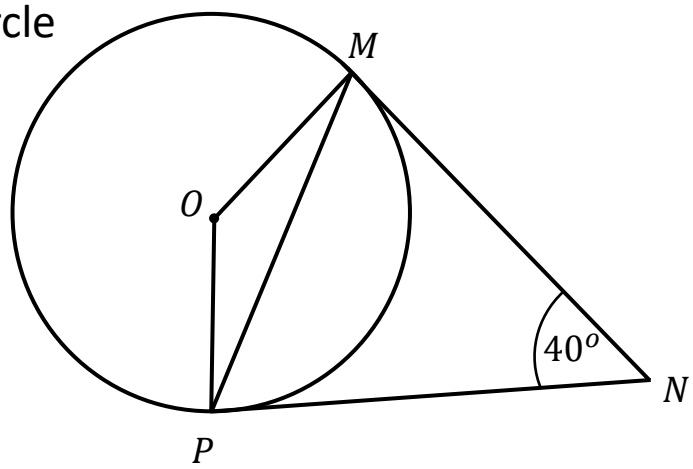
Z, X and Y are points on the circumference of the circle, centre O
Work out the size of angle XOY



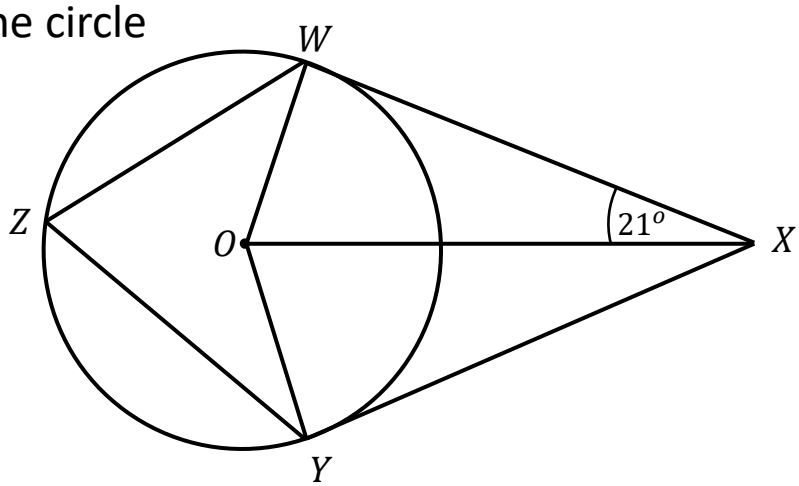
M and P are points on the circumference of the circle, centre O
Work out the size of angle NOM



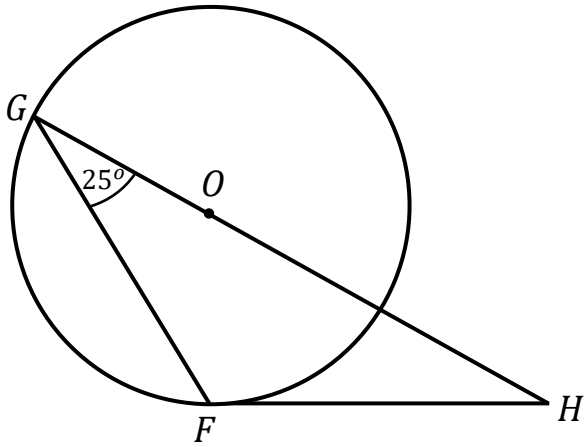
M and P are points on the circumference of a circle, centre O
MN and PN are tangents to the circle
Angle $MNP = 40^{\text{o}}$
Find the size of angle OPM



Z, W and Y are points on the circumference of a circle, centre O
WX and YX are tangents to the circle
Angle $\text{WXO} = 21^{\text{o}}$
Find the size of angle WZY

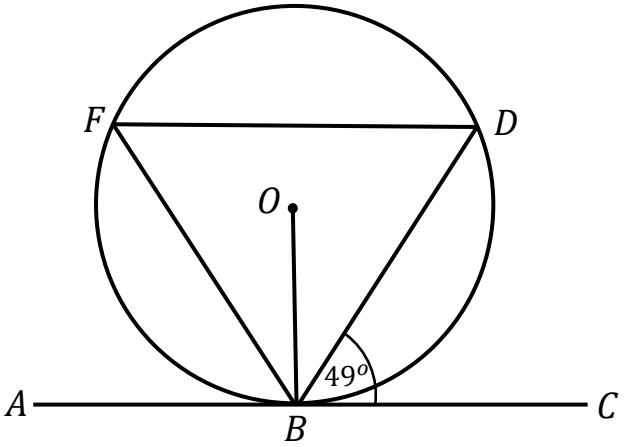


G and F are points on the circumference of a circle, centre O
FH is a tangent to the circle
Angle $\mathrm{FGH} = 25^{\mathrm{o}}$
Find the size of angle GHF

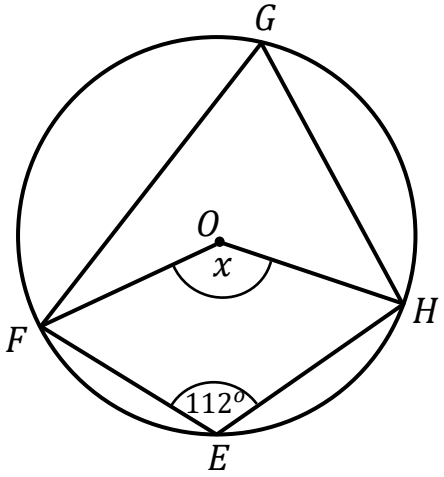


B, D and F are points on the circumference of a circle, centre O
ABC is a tangent to the circle
Angle $CBD = 49^{\mathrm{o}}$

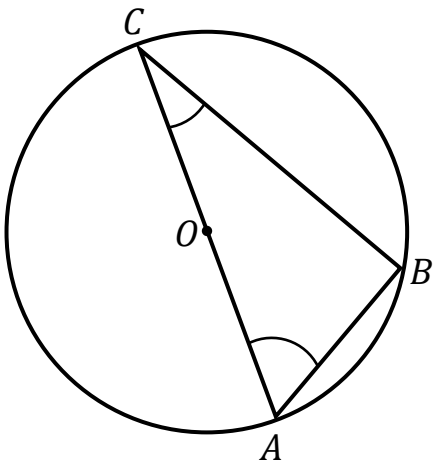
i) Find the size of angle BFD
ii) Find the size of angle OBD



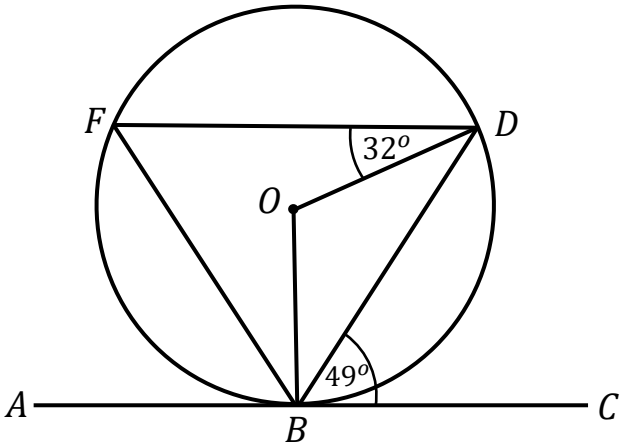
F, G, H and E are points on the circumference of the circle, centre O
Angle $FEH = 112^o$
Find the size of angle x



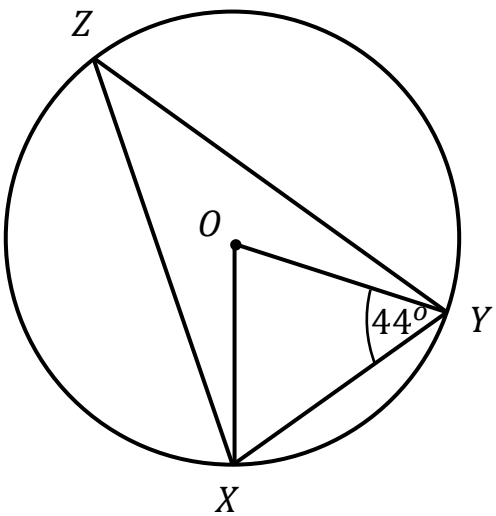
A, B and C are points on the circumference of a circle, centre O
Angle BCA and angle BAC are in the ratio $4:5$
Find the size of angle BCA and angle BAC



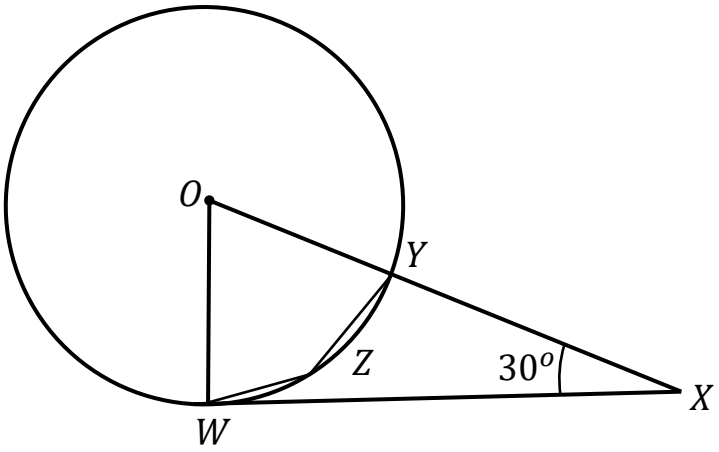
$F, D, and\ B$ are points on the circumference of the circle, centre O
ABC is tangent to the circle
Angle $FDO = 32^o$
Angle $DBC = 49^o$
Find the size of angle FBO



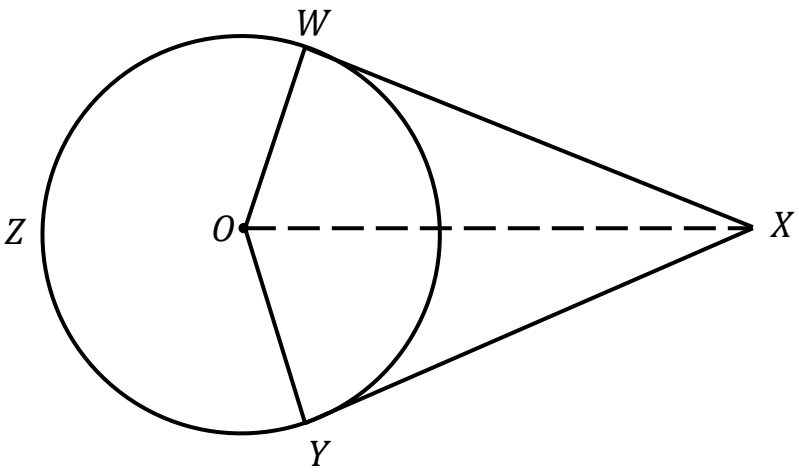
$Z, X, and\ Y$ are points on the circumference of the circle, centre O
Angle $XYO = 44^o$
Find the size of angle XZY



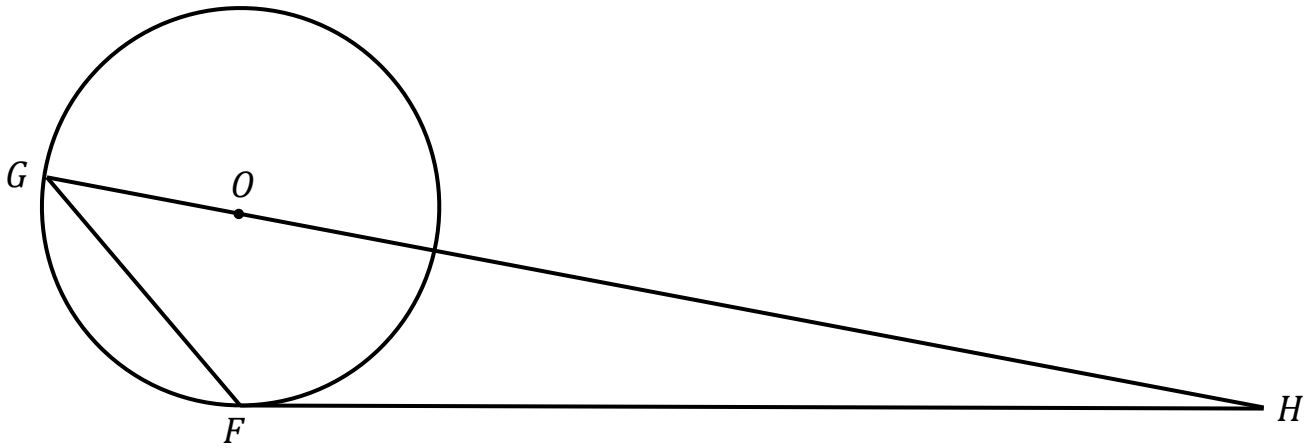
W, Y and Z are points on the circumference of a circle, centre O
OYX is a straight line and WX is a tangent to the circle
Given that angle $OXW = 30^o$ find the size of angle WZY



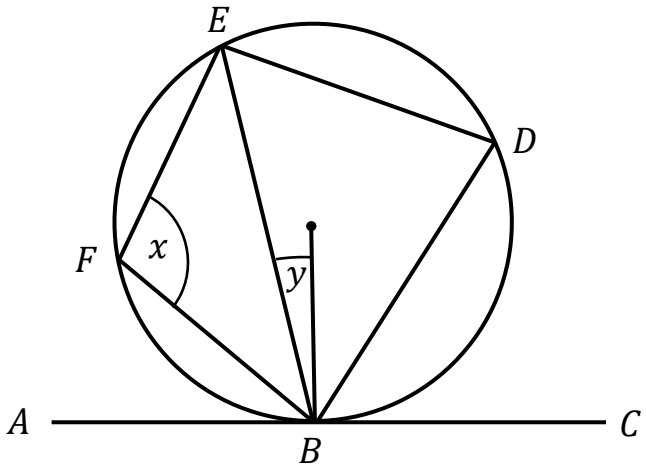
W, Z and Y are points on the circumference of the circle, centre O
OW and OY are both radii of length 10cm
WX and YX are both tangents to the circle
Given that $OX = 20cm$, find the arc length WZY



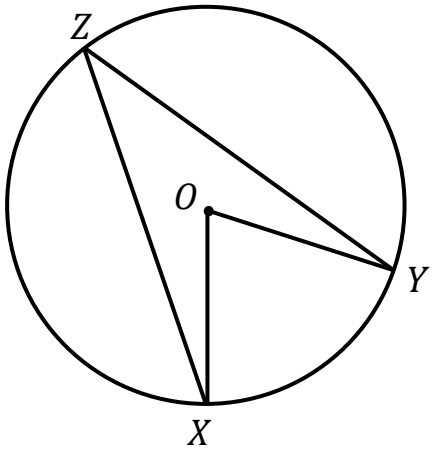
G and F are points on the circumference of the circle, centre O
FH is tangent to the circle
GOF is a straight line
Angle $FGO = y^o$
Find the size of angle GHF in terms of y



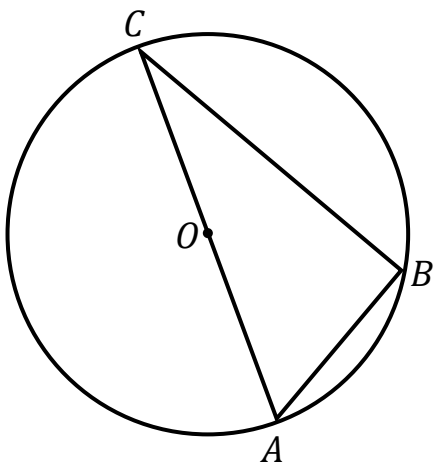
$F, E, D, and\ B$ are points on the circumference of the circle, centre O
ABC is tangent to the circle
Show that $x - y = 90$



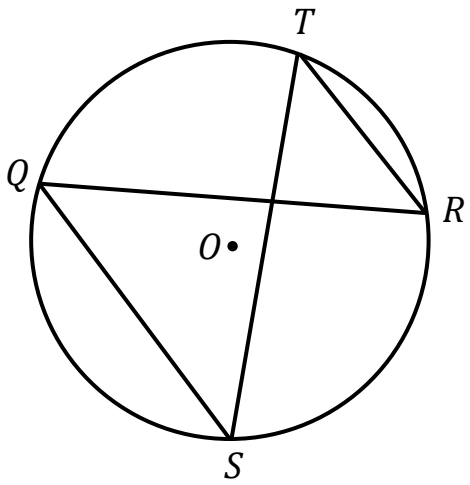
X, Y and Z are points on the circumference of a circle, centre O
Prove that angle XOY is twice that of angle XZY
Do not use any circle theorems in your proof



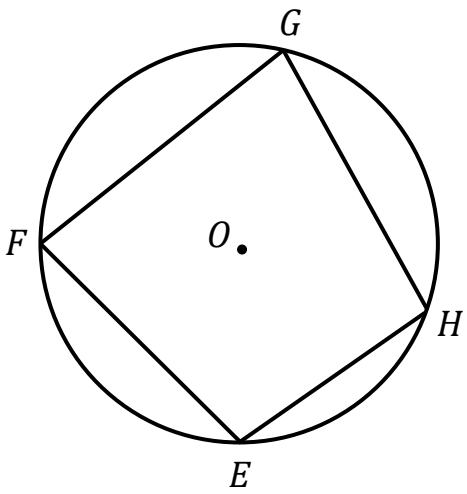
A, B and C are points on the circumference of a circle, centre O
COA is a diameter of the circle
Prove that angle ABC is 90^o
Do not use any circle theorems in your proof



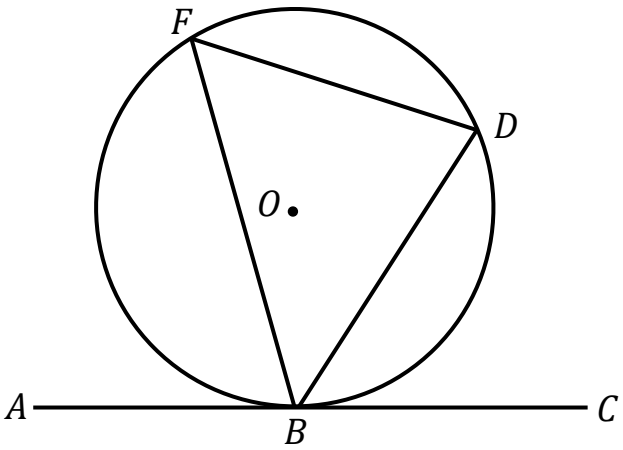
T, R, S and Q are points on the circumference of a circle, centre O
Prove that angle SQR and angle RTS are equal



E, F, G and H are points on the circumference of a circle, centre O
Prove that angle GFE and angle GHE sum to 180^o



B, D and F are points on the circumference of a circle, centre O
ABC is a tangent to the circle
Prove that angle CBD and angle BFD are equal



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Week 11

Vector Proofs

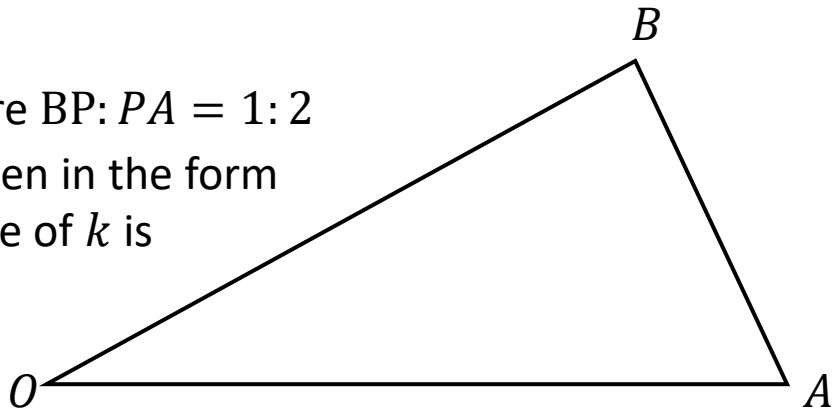
- Show that vectors are parallel
- Show that vectors are collinear

$\overrightarrow{OA} = \boldsymbol{a}$

$\overrightarrow{OB} = \boldsymbol{b}$

P is the point on BA, where BP: $PA = 1:2$

Show that $\overrightarrow{OP}$ can be written in the form $k(\boldsymbol{a} + 2\boldsymbol{b})$ where the value of k is to be found

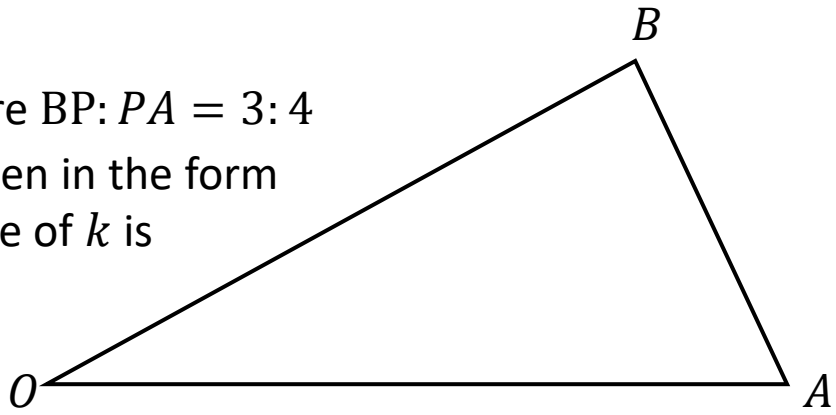


$\overrightarrow{OA} = 2\boldsymbol{a}$

$\overrightarrow{OB} = 3\boldsymbol{b}$

P is the point on BA, where BP: $PA = 3:4$

Show that $\overrightarrow{OP}$ can be written in the form $k(\boldsymbol{a} + 2\boldsymbol{b})$ where the value of k is to be found



$ABCD$ is a trapezium

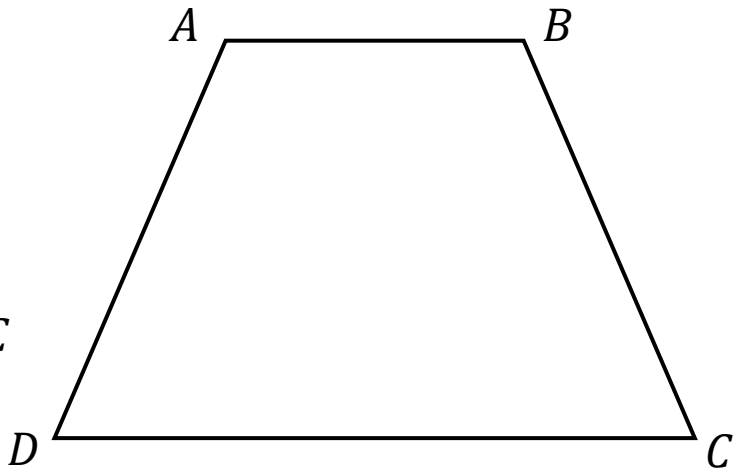
$\overrightarrow{DA} = \mathbf{3a}$

$\overrightarrow{AB} = \mathbf{3b}$

$\overrightarrow{DC} = 2\overrightarrow{AB}$

M is the midpoint of DB

Show that AM is parallel to BC



$ABCDEF$ is a regular hexagon with centre O

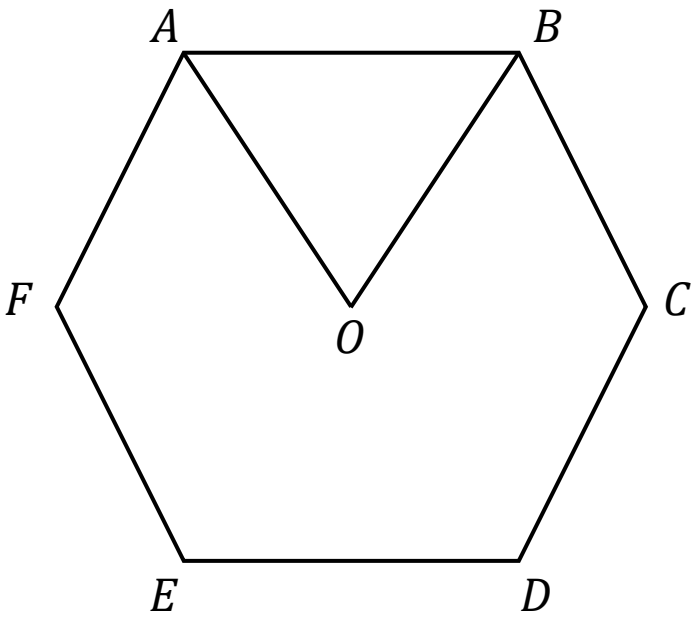
$\overrightarrow{OA} = \boldsymbol{a}$

$\overrightarrow{OB} = \boldsymbol{b}$

M is the midpoint of CD

ED has been extended to the point G where $ED : DG = 3 : 2$

Prove that A, M and G are on the same straight line



$OABC$ is a parallelogram

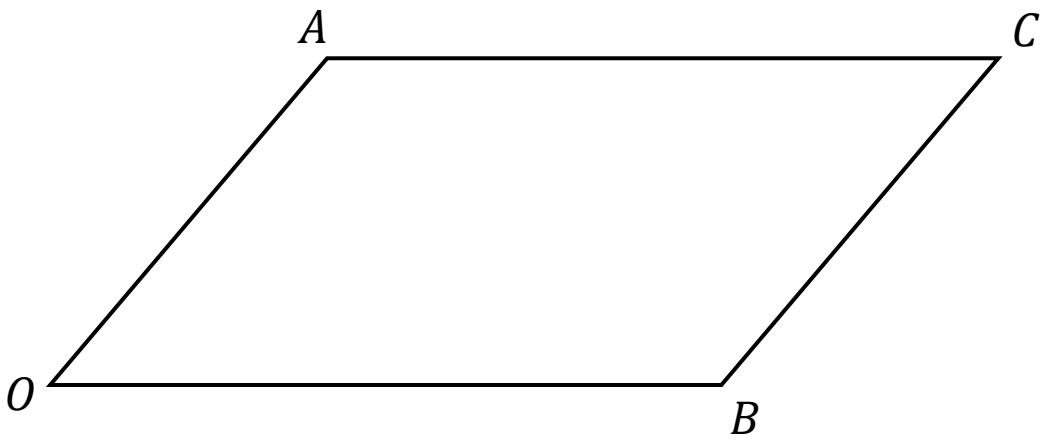
$\overrightarrow{OA} = \boldsymbol{a}$

$\overrightarrow{OB} = \boldsymbol{b}$

P is the point on OC where $OP : PC = 2 : 1$

M is the midpoint of BC

Prove that A, P and M are on the same straight line



$OABC$ is a parallelogram

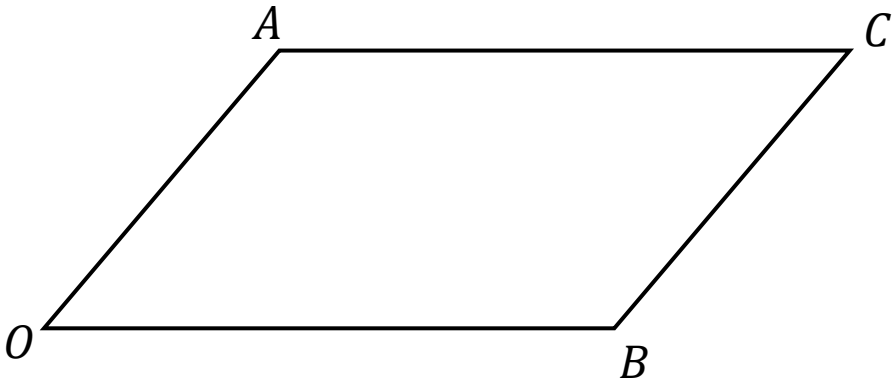
$\overrightarrow{OA} = 3\boldsymbol{a}$

$\overrightarrow{OB} = 4\boldsymbol{b}$

M is the midpoint of AB

OBD is a straight line where $OB : BD = 2 : 5$

Given that $\overrightarrow{MD} = k\boldsymbol{b} - \frac{3}{2}\boldsymbol{a}$ find the value of k



$\overrightarrow{OA} = 2\boldsymbol{a}$

$\overrightarrow{OB} = 2\boldsymbol{b}$

C is the midpoint of OA

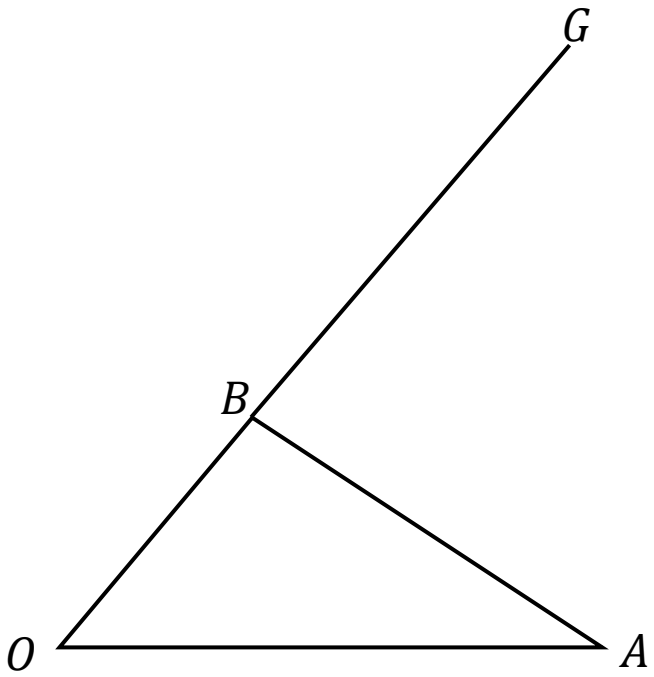
$\overrightarrow{OB} = \frac{2}{5}\overrightarrow{OG}$

F is a point on BA

$\overrightarrow{BF} = k\overrightarrow{BA}$

CFG is a straight line

Find the value of k



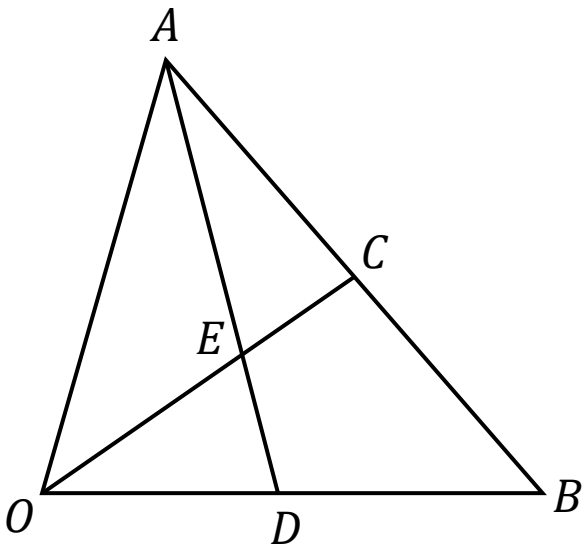
$\overrightarrow{OA} = \boldsymbol{a}$

$\overrightarrow{OB} = \boldsymbol{b}$

C is the midpoint of AB

$OE : EC = 4 : 3$

Work out the ratio of $OD : DB$



$ABCD$ is a quadrilateral

$\overrightarrow{AB} = \boldsymbol{a}$

$\overrightarrow{BC} = \boldsymbol{b}$

$\overrightarrow{DA} = \boldsymbol{a} - \boldsymbol{b}$

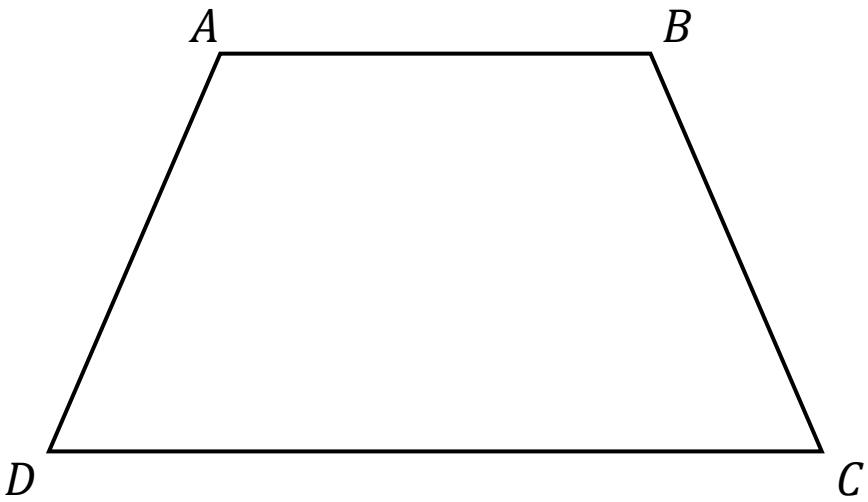
E is the midpoint of BC

F lies on the point DE

$DF : FE = n : 1$

AFC is a straight line

Find the value of n



$OABC$ is a parallelogram

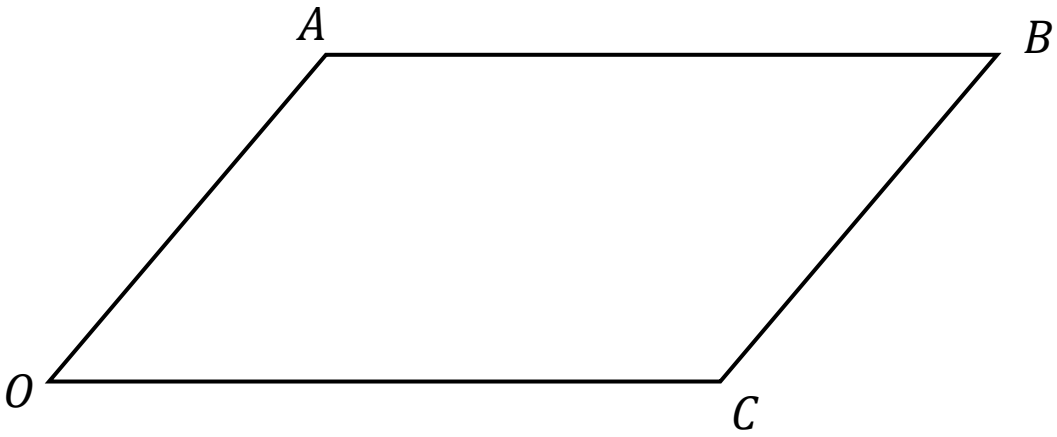
$\overrightarrow{OC} = \boldsymbol{c}$

$\overrightarrow{OB} = \boldsymbol{b}$

D is the point on OC where $OD : DC = 1 : 3$

E is the point on OB where $OE : EB = 1 : 2$

Work out, in its simplest form, the ratio $AD : AE$



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Week 12

Parallel/Perpendicular Lines
Equations of Tangents

- Apply properties of parallel and perpendicular lines to find unknown straight-line equations
- Find the equation of a tangent line

Find the equation of the straight line passing through $(2, -3)$ parallel to $4x + 2y = 8$ giving your answer in the form $ay + bx + c = 0$

Line A passes through $(0,3)$ and $(-2,8)$
Line B is perpendicular to line A and passes through $(5, -5)$
Find the equation of line B giving your answer in the form $ay + bx + c = 0$

Line A has equation $3x - 5y - 20 = 0$

i. Find the gradient of the line

ii. Line A intersects the x-axis at F and the y-axis at G find the midpoint of FG

Line A passes through $(2,8)$ and $(4,5)$
Line B is the perpendicular bisector of Line A
Find the equation of Line B in the form $ax + by = c$

A line of gradient -4 passes through the points $(-4,7)$ and $(a,5)$
Find the value of a

A line passes through $(1,5)$ and $(5,7)$
Another line passes through $(-1,7)$ and $(2,a)$
Find the value of a if:

i. The lines are parallel
ii. The lines are perpendicular

A triangle ABC has points $A(3,11), B(-9,7)$ and $C(-7,1)$
Prove that the angle ABC is a right angle

A circle has centre $(3,6)$
The point $A(11,9)$ lies on the circumference of the circle
Find the equation of the tangent to the circle at A

A circle has equation $x^2 + y^2 = 17$
Point $P(1,4)$ lies on the circle
Find the equation of the tangent to the circle at point P

Line A is tangent to the circle $x^2 + y^2 = 13$ at point $P(2,3)$
Line A crosses the y-axis at point F
Find the area of triangle OPF

Line A has equation $2y + 4x = 6$
Line B passes through $(5,6)$ and is perpendicular to Line A
Line B crosses the x-axis at F and the y-axis at G
Given that O is the origin, find the area of triangle OFG

Line A has equation $2x + 3y = 26$
Line B passes through the origin O and is perpendicular to Line A
Line A intersects the x-axis at F
Line A and Line B intersect at C
Find the area of triangle OFC